Q1: Prove that the set of all ordered pairs (x_1, x_2) of the elements of field of real no.s forms a vector space w.r.t addition and scalar multiplication defined as.

$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$

$c \odot (x_1, x_2) = (cx_1, cx_2)$

Solu^n: V has element of the type (x_1, x_2), $x_1, x_2 \in \mathbb{R}$

To be a vector space $(V, \oplus, \odot)$ should satisfy the following conditions.

1. $(V, \oplus)$ is an abelian gp.
2. $(V, \oplus, \odot)$ satisfies the five properties of scalar multiplication.

$(V, \oplus)$ is an abelian group.

1. (i) $(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, x_2 + y_2) \in \mathbb{R}$ $\forall x_1, x_2, y_1, y_2 \in \mathbb{R}$

$\therefore$ It is closed under vector addition.

(ii) $((x_1, x_2) \oplus (y_1, y_2)) \oplus (w_1, w_2)$

$= (x_1 + y_1, x_2 + y_2) \oplus (w_1, w_2) = (x_1 + y_1 + w_1, x_2 + y_2 + w_2)$

$= (x_1 + (y_1 + w_1), x_2 + (y_2 + w_2)) = (x_1, x_2) \oplus ((y_1, y_2) \oplus (w_1, w_2))$

$\therefore$ Associativity holds.

(iii) Let $e = (e_1, e_2)$ be the identity element.

$(x_1, x_2) \oplus (e_1, e_2) = (x_1, x_2)$ $\quad \forall (x_1, x_2) \in V$, $x_1, x_2 \in \mathbb{R}$

$\Rightarrow (x_1 + e_1, x_2 + e_2) = (x_1, x_2)$

$\Rightarrow x_1 + e_1 = x_1$ & $x_2 + e_2 = x_2$

$\Rightarrow e_1 = 0$ & $e_2 = 0$

$\therefore (0, 0)$ is an identity element.